Rheological properties of paraffin as an analogue material for viscous crustal deformation

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Abstract

The rheological stratification induced by temperature variations within the crust requires analogue materials with temperature-sensitive viscosity to properly simulate the change of mechanical properties with depth. Rheological calibration experiments on commercial paraffin wax with melting point of $53 \pm 1°C$ show that this material is a nonlinear fluid ($1.6 \leq n \leq 2.1$) for homologous temperatures $T/T_m < 0.70$, and an approximately Newtonian fluid ($1.0 \leq n \leq 1.3$) for $T/T_m \geq 0.70$. The decrease in the stress exponent n is probably associated with the $\alpha–\beta$ phase transition of paraffin. Effective viscosity values are strongly temperature dependent, varying by nine orders of magnitude from 10^{10} to 10 Pa s in the temperature range between 30 and 52°C. The rheological properties of paraffin make this material useful for modelling crustal rheology and tectonic processes, when linear dimensions and time scale down by a factor of 10^6 and 10^{11}, respectively. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

For temperatures $T \geq 1/2 \; T_m$ (where T_m is the solidus temperature) and at geological time scales, rocks exhibit a change in deformation mechanism from frictional Coulomb plasticity to steady-state ductile flow (e.g. Ranalli, 1995, and references therein). Thus, Coulomb brittle rheologies are representative of shallow crustal conditions, whereas steady-state thermally activated creep is dominant at depth. Consequently, to simulate properly the crustal rheological stratification in laboratory analogue models, it is necessary to take into account the variations in mechanical properties induced by temperature. This has been done in two different ways, i.e. (i) using different materials, such as sand and silicone putty, for brittle and ductile behaviour, respectively (e.g. Davy and Cobbold, 1991, and references therein); or (ii) using a single material with temperature-sensitive viscosity (Cobbold and Jackson, 1992). The second approach has the advantage of reproducing the mechanical consequences of thermal readjustment during the model run. This seems to be particularly important in the experimental investigation of lithosphere stretching and of the stability of mountain belts.

We have investigated the rheological properties of a 52/54 EN type commercial paraffin (producer: INA–Naftaplin, Zagreb, Croatia), determined as a function of the homologous temperature, T/T_m. Our results show that paraffin behaves as a nearly Newtonian material for homologous temperatures higher than 0.7. Paraffin is a useful analogue material satisfying scaling requirements for modelling ductile crustal deformation using variations in viscosity in a small temperature range.

2. Creep in the continental crust

In general terms, flow of polycrystalline aggregates is related to stresses by a constitutive equation (see e.g. Ranalli, 1995)

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$$R(\sigma_{ij}, \dot{\varepsilon}_{ij}, M_i, S_i) = 0 \tag{1}$$

where σ_{ij}, $\dot{\varepsilon}_{ij}$ are stress and strain rate tensors, and M_i, S_i are material parameters and state variables, respectively.

For creep in the mantle and in the mid-to-lower crust the flow law may be written in the form (see Ranalli, 1995, and references therein)

$$\dot{\varepsilon} = A(\sigma_1 - \sigma_3)^n \, d^{-m} \exp(-E/RT), \tag{2}$$

where $\dot{\varepsilon}$ is the longitudinal strain rate associated with the stress difference $\sigma_1 - \sigma_3$, E is the activation energy for creep, R is the universal gas constant, T (K) is the absolute temperature, d is the grain size, n and m are stress and grain size exponents, respectively, and A is a material constant. For fine grained aggregates ($d \leq 100$ μm as an order of magnitude), linear or Newtonian creep ($n = 1$, $m = 2$–3) is predominant, while in coarser-grained aggregates and at higher stress level, non-linear creep is observed ($n = 2$–4, $m = 0$; see, among others, Kirby, 1983; Carter and Tsenn, 1987; Ord and Hobbs, 1989; Ranalli, 1995).

Average crustal rock compositions are adequately represented by quartz-rich or feldspar-rich rocks. In these rocks, for $d = 10$–100 μm and $\sigma \geq 1$–10 MPa, n ranges between 2 and 3.5 and $m = 0$. The activation energy E is usually between 150 and 250 kJ mol^{-1} (Ranalli, 1995, and references therein), resulting in effective viscosities for the ductile lower crust varying from 10^{17} to 10^{21} Pa s.

3. Experimental material and results

Paraffin wax has been commonly used as an analogue material in reproducing small-scale structures such as folds (e.g. Cobbold, 1975; Abbassi and Mancktelow, 1992; Grujic and Mancktelow, 1995), boudins (Neurath and Smith, 1982) and shear zones (e.g. Hafner and Passchier, 1997), and also to investigate large-scale processes such as mantle convection (Jacoby, 1976), subduction (Shemenda, 1994), and continental extension (Brune and Ellis, 1997).

The rheology of commercial paraffin waxes was previously investigated by Cobbold (1975), Neurath and Smith (1982), Mancktelow (1988) and Grujic and Mancktelow (1995). These studies indicate that paraffin waxes exhibit power-law creep for relatively low homologous temperatures ($0.5 \leq T/T_m \leq 0.6$). The stress exponent n decreases from 4.1 to 2.4 with increasing paraffin melting point from 42 to 52°C (Mancktelow, 1988). The power-law field corresponds to homologous temperatures below the α–β phase transition of paraffin (Freund et al., 1982; Mancktelow, 1988). The α–β phase transition temperature, to which corresponds a marked weakening of paraffin waxes (Mancktelow, 1988), is a function of the paraffin melting point (for details see Freund et al., 1982).

We used a commercial refined macrocrystalline paraffin wax of 52/54 EN type, consisting of a mixture of saturated hydrocarbons (alcanes) of general composition C_nH_{2n+2} with melting point at 53 ± 1°C. The average molecular weight of the components is 350, the density is 0.81 g cm^{-3} at $T = 15$°C, and the thermal diffusivity k is 8×10^{-8} m^2 s^{-1}. It contains 0.7 wt% oil and does not contain substances detrimental to health. Using systematics collected from Freund et al. (1982) and Mancktelow (1988) the α–β transition in our paraffin should occur at about 35°C.

Rheological calibration was performed at temperatures between 29 and 40°C ($0.55 \leq T/T_m \leq 0.75$) by applying a series of constant strain rates to the paraffin samples and measuring the change of the differential stress as the strain progresses. In the temperature range from 29 to 35°C, flow properties were determined in uniaxial compression tests, performed in a triaxial testing machine adapted for the purpose. Moulded paraffin cylinders (5 cm in diameter and 10 cm high) were maintained at 29 ± 0.1°C and 35 ± 0.1°C by a water bath and shortened at constant strain rate (5×10^{-6} s$^{-1} \leq \dot{\varepsilon} \leq 5 \times 10^{-5}$ s^{-1}). The stress difference is
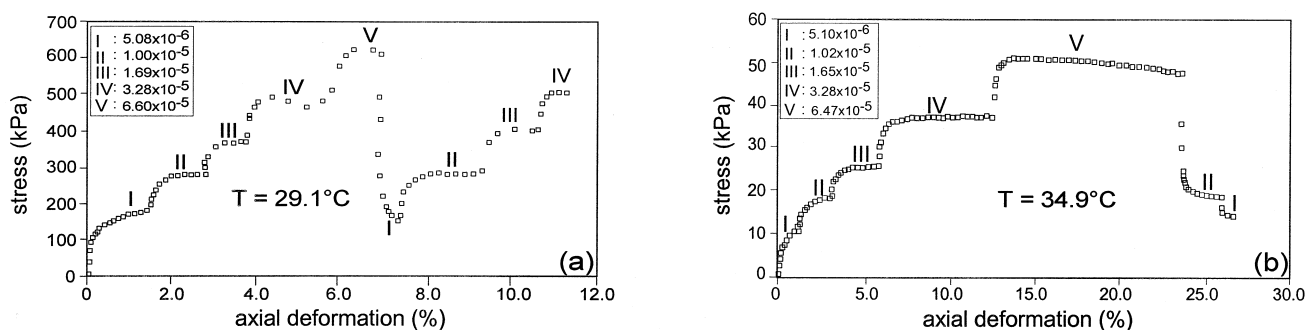

Fig. 1. Plots of differential stress vs % strain for uniaxial compression tests on paraffin cylinders (a) at $T = 29$°C and (b) $T = 35$°C. Roman numerals denote strain rates at different test stages.
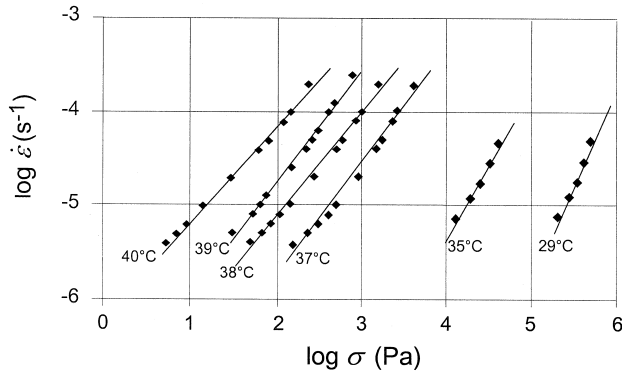
Fig. 2. Log–log plot of strain rate ($\dot{\varepsilon}$) vs stress (σ) at different temperatures. Best-fit lines are also shown. See also Table 1.

computed from the applied uniaxial load, corrected by the hydrostatic pressure, and the instantaneous value of the cross-sectional area of the cylinder. The strain rate is obtained by dividing the ram speed by the instantaneous cylinder height. During a single test, the strain rate was changed to obtain further information about the stress–strain relations. For constant strain rate, plots of differential stress against per cent shortening (Fig. 1a and b) show a nearly steady-state creep after yield, with a small amount of strain softening in some cases. In the field of steady-state flow, the flow stress was estimated by linear fitting of the stress data. The stepped strain rate experiments demonstrate a good reproducibility of the stress levels at varying strain rates. These results are in agreement with those obtained by Neurath and Smith (1982) for different paraffin waxes.

For temperatures between 37 and 40°C, paraffin cylinders collapsed under their own weight and consequently the uniaxial test could not be performed. Thus, rheological calibration was achieved using a rheometer (Rheometrics Scientific RMS800). In this apparatus the flow properties of the paraffin specimens were evaluated, for each 1°C-temperature increase, under an annular shear stress transmitted using a

Table 1
Stress exponent n at different temperatures and homologous temperatures

Experiment	T (°C)	T/T_m	n
uniaxial test	29	0.55	2.11
uniaxial test	35	0.66	1.59
RSM800 test	37	0.70	1.32
RSM800 test	38	0.72	1.13
RSM800 test	39	0.73	1.21
RSM800 test	40	0.75	1.00

plane-parallel plates geometry rotor (8 mm in diameter), for 4×10^{-6} s$^{-1} \leq \dot{\varepsilon} \leq 4 \times 10^{-4}$ s^{-1}.

Since only the stress dependence of the strain rate at any given temperature was measured in both tests, the rheological Eq. (2) can be written as

$$\dot{\varepsilon} = A_0 \sigma^n, \qquad (3)$$

where A_0 is a T-dependent parameter. The experimental data, when plotted on logarithmic coordinates, fall on straight lines (Fig. 2). A standard regression then yields the stress exponent n as the slope of the best-fit lines. Results show that the material is nonlinear ($1.59 \leq n \leq 2.11$) for low homologous temperatures ($T/T_m < 0.70$) and approximately Newtonian ($1.00 \leq n \leq 1.32$) for $T/T_m \geq 0.70$. The fit is usually very good, with regression coefficient larger than 0.9. Results are shown in Table 1.

As suggested by Neurath and Smith (1982), effective viscosity (η_{eff}) in the uniaxial tests is computed using the relation $\eta_{eff} = 1/3 \, (\sigma/\dot{\varepsilon})$. In the rheometer tests the viscosity is calculated from the relation $\eta = \sigma/2\dot{\varepsilon}$. In addition, at higher temperatures paraffin viscosity was also determined using a torsion viscometer (Brookfield model RV). Viscosity values vary as a function of the temperature from about 10^{10} Pa s at 29°C, to about 10 Pa s at 52°C. In the Newtonian field ranging from 37 to 40°C, the viscosity is practically independent of stress, and ranges between 10^7 and 10^5 Pa s, as orders of magnitude (Fig. 3).
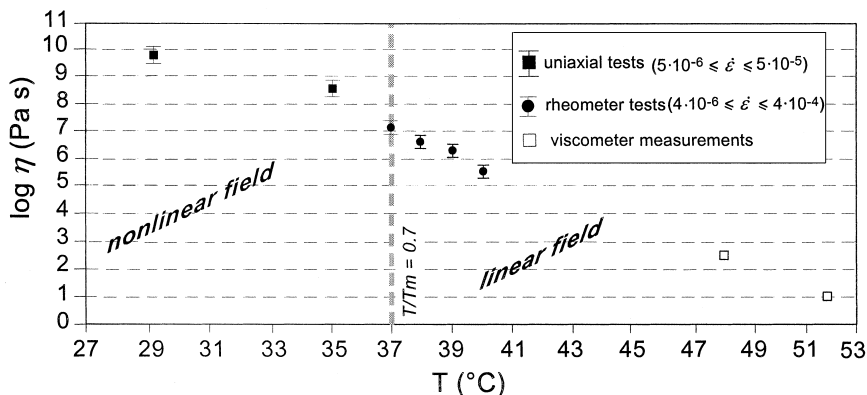

Fig. 3. Viscosity of paraffin as a function of temperature in the nonlinear ($T/T_m < 0.7$) and linear ($T/T_m > 0.7$) range.
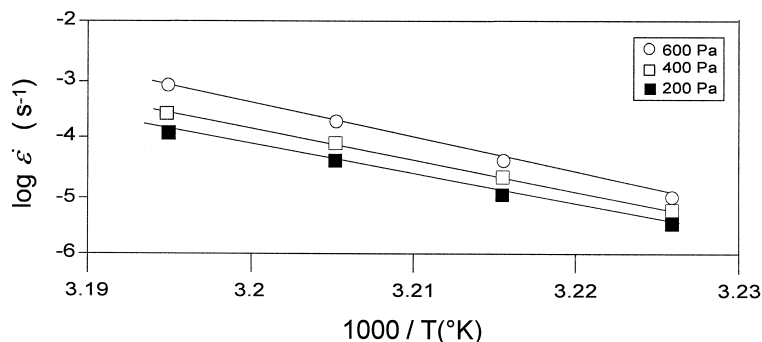
Fig. 4. Log plot of strain rate ($\dot{\varepsilon}$) vs reciprocal temperature ($1000/T$) for the rheometer tests.

In consequence of the exponential dependence of strain rate upon reciprocal temperature according to Eq. (2), plots of log $\dot{\varepsilon}$ vs $1/T$ are linear, and allow the determination of the activation energy E from the slope of the best-fit lines (Fig. 4). The number of data points is insufficient to estimate E in the nonlinear field. In the linear field, activation energy values range between 950 and 1100 kJ mol^{-1}. Also in this case the fit is good, with regression coefficient larger than 0.9. The E values are considerably higher than those obtained by Mancktelow (1988) for the β-phase. A high activation energy implies a strong temperature dependence, which is an advantage for analogue materials.

4. Scaling for viscous crustal deformation

Scaling theory requires that geometrical, kinematic and dynamic similarity exists between the analogue model and the natural prototype (Ramberg, 1981; Weijermars and Schmeling, 1986). In order to simplify the scaling procedure, lower crustal rocks are assumed to behave as a viscous fluid (e.g. Davy and Cobbold, 1991). This allows determination of scale factors using the parameters of paraffin in the Newtonian range. Temperature is not considered in the scaling procedure, but variations in temperature are used to control the mechanical behaviour and the strength gradient of the model.

For negligible inertial forces, which is the case for many geological processes, scale model theory for Newtonian flow requires geometrical similarity to achieve dynamic similarity between the natural prototype (N) and the model (M) (Weijermars and Schmeling, 1986). This yields a set of dimensional scaling rules involving the three independent parameters density (ρ), length (l), and gravitational acceleration (g) (Weijermars and Schmeling, 1986; Davy and Cobbold, 1991). For paraffin under a natural gravity field, assuming a nature vs model length ratio (l_N/l_M) of 10^6, and crustal rocks density of 2.7 g cm^{-3}, we obtain the following stress scale factor:

$$\sigma_N/\sigma_M = (\rho_N g_N l_N/\rho_M g_M l_M) \sim 10^6. \tag{4}$$

To set the thermal time scale (t) we use the following expression, derived from the non-dimensionalization of the equation of heat transfer in the crust (Davy and Cobbold, 1991; Cobbold and Jackson, 1992):

$$t_N/t_M = \{(l_N)^2/k_N\}/\{(l_M)^2/k_M\}. \tag{5}$$

For a standard crustal diffusivity $k_N = 10^{-6}$ m^2 s^{-1}, Eq. (5) yields an experimental time scale factor for paraffin of the order of 10^{11}. Considering a strain rate of 10^{-14}–10^{-16} s^{-1}, valid for orogenic processes as a whole, the average experimental strain rate should therefore be of the order of 10^{-3}–10^{-5} s^{-1}.

For an experimental strain rate of 10^{-4}–10^{-5} s^{-1}, viscosity values of paraffin in the Newtonian range are associated with a flow stress drop of two orders of magnitude (from 10^3–10^2 to 10–1 Pa) for a 3°C-temperature increase. The corresponding stress values for the natural prototype, as defined by the stress scale of Eq. (4), thus vary from 10^3–10^2 to 1–10 MPa. These values are in good agreement with the creep strength values of crustal rocks at temperatures above the brittle–ductile transition (see e.g. Ranalli, 1995).

5. Conclusions

Rheological calibration experiments on commercial paraffin wax with $T_m = 53 \pm 1$°C have shown that the behaviour changes gradually from nonlinear to Newtonian with increasing homologous temperature. At $T/T_m = 0.55$ ($T = 29$°C) the stress exponent is relatively large ($n > 2$). As T/T_m increases, n decreases and approaches unity. At $T/T_m > 0.70$ ($T > 37$°C), the behaviour is practically Newtonian.

The activation energy in the linear range is high ($E = 950$–1100 kJ mol^{-1}). This high temperature sensitivity results in a strong temperature dependence of viscosity. The linear viscosity varies by two orders of magnitude within the narrow experimental temperature range (37–40°C). The scaling requirements for ana-

logue modelling of viscous flow in the crust are satisfied for experimental length scales of 10^6 and experimental strain rates in the range 10^{-5}–10^{-4} s^{-1}.

Acknowledgements

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